Exercise 32

Match the functions f with the plots of their gradient vector fields labeled I–IV. Give reasons for your choices.

$$f(x, y) = \sin \sqrt{x^2 + y^2}$$

Solution

Take the gradient of f.

$$\begin{aligned}
\nabla f &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \\
&= \left\langle \left(\cos \sqrt{x^2 + y^2}\right) \frac{\partial}{\partial x} \sqrt{x^2 + y^2}, \left(\cos \sqrt{x^2 + y^2}\right) \frac{\partial}{\partial y} \sqrt{x^2 + y^2} \right\rangle \\
&= \left\langle \left(\cos \sqrt{x^2 + y^2}\right) \frac{x}{\sqrt{x^2 + y^2}}, \left(\cos \sqrt{x^2 + y^2}\right) \frac{y}{\sqrt{x^2 + y^2}} \right\rangle
\end{aligned}$$

The vector field is radially symmetric and changes direction sinusoidally. This matches with plot I.